

**[06-09-08-T10]***Solving compound inequalities*

A compound inequality is several inequalities joined by the logical operators "and" or "or". The logical operator "and" is often represented by the symbol " $\wedge$ ". The "or" is often represented by " $\vee$ ".

[EX1] Find all values of  $x$  for which the following is true.  $x + 3 < 7$  and  $x + 3 > 1$ .

Solution.

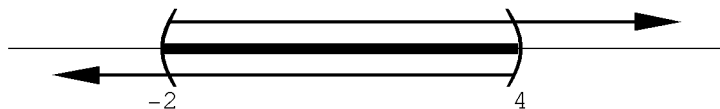
$$x + 3 < 7 \implies x < 4 \implies x \in (-\infty, 4)$$

$$x + 3 > 1 \implies x > -2 \implies x \in (-2, \infty +)$$

$$(x + 3 < 7 \text{ and } x + 3 > 1) \implies x \in (-\infty, 4) \cap (-2, \infty +)$$

$$\therefore x \in (-2, 4)$$

Note: a graph typically makes determining  $(-\infty, 4) \cap (-2, \infty +)$  easy. In this example,



[EX2] Find all values of  $x$  for which the following is true.  $-3 < x + 5 < 7$ .

$-3 < x + 5 < 7$  means two inequalities connected by "and".

That is  $-3 < x + 5 < 7 \iff -3 < x + 5$  and  $x + 5 < 7$ . As such, they can be split apart and handled as in [EX1]. They can also be worked, as follows, without splitting them apart.

$$-3 < x + 5 < 7$$

$$\iff -3 - 5 < x < 7 - 5$$

$$\iff -8 < x < 2$$

$$\therefore x \in (-8, 2)$$

[EX3] Find all values of  $x$  for which the following is true.  $4 < 2x + 5 < 15$ .

Solution.

$$4 < 2x + 5 < 15$$

$$\iff -1 < 2x < 10$$

$$\iff -\frac{1}{2} < x < 5$$

$$\therefore x \in \left(-\frac{1}{2}, 5\right)$$

■ Sometimes, as the next example shows, the compound inequalities must be split apart.

[EX4] Find all values of  $x$  for which the following is true.  $4x < 2x < 15$ .

Trying to work them together, leads to various dead ends.

$$4x < 2x < 15 \iff 2x < x < \frac{15}{2} \quad \text{Bust!}$$

$$4x < 2x < 15 \iff 4x - 2x < 0 < 15 - 2x \quad \text{Bust!}$$

Solution.

Splitting apart the inequalities works well.

$$4x < 2x < 15 \iff (4x < 2x \wedge 2x < 15). \text{ Then,}$$

$$(4x < 2x \wedge 2x < 15) \iff (x < 0 \wedge x < \frac{15}{2})$$

$$\therefore x \in (-\infty, 0)$$

[EX5] Find all values of  $x$  for which the following is true.  $3x + 5 \leq 7$  or  $5x + 2 > 9$ .

Solution.

$$3x + 5 \leq 7$$

$$\iff 3x \leq 2$$

$$\iff x \leq \frac{2}{3}$$

$$5x + 2 > 9$$

$$\iff 5x > 7$$

$$\iff x > \frac{7}{5}$$

$$3x + 5 \leq 7 \vee 5x + 2 > 9 \iff x \leq \frac{2}{3} \vee x > \frac{7}{5}$$

$$\therefore x \in (-\infty, \frac{2}{3}] \cup (\frac{7}{5}, \infty +)$$